Fluxes and nuclear abundances of cosmic rays inside the magnetosphere using a transmission function approach

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Abstract

At 1 AU and outside the Earth’s magnetosphere, the relative abundances to protons for He (He/p), C (C/p) and Fe (Fe/p) nuclei were calculated using the observation data of AMS-01 (for p and He) and HEAO-3 (for C and Fe) above 0.8 GeV/nucleon. In addition, the transmission function (TF) for the GCR propagation inside the magnetosphere was evaluated using the IGRF and T96 (introduced by Tsyganenko and Stern) models to obtain permitted and forbidden trajectories inside the magnetosphere. The TF allowed one to derive the primary He-nuclei fluxes in the same geomagnetic regions of AMS-01 observations. These fluxes were found in good agreement with the observation data. Furthermore inside the magnetosphere in addition to the flux of helium, it allowed one to obtain those of the primary p, C, and Fe nuclei and the relative abundances of He, C and Fe nuclei to protons from the same observation data of AMS-01 and HEAO-3 above 0.8 GeV/nucleon. Up to a geomagnetic latitude of 45.84°, the relative isotopic abundances were found to depend on the mass number I and, on average, range from a factor 2.31 up to 3.35 larger than those outside the magnetosphere at 1 AU. Thus, the magnetospheric isotopic/nuclear relative abundances differ from those inside the solar cavity and those in the interstellar space. The usage of the TF approach can allow one to determine the nuclear abundances in the magnetosphere at any geomagnetic latitude and, thus, any orbit, provided that the CR spectra are determined at 1 AU.

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1. Introduction

Galactic cosmic rays (GCR’s) are the dominant component of the charged particles present in space above few hundreds MeV of kinetic-energy. The knowledge of their relative abundances is based on measurements performed by several satellites, most of them operating outside the magnetosphere, and by stratospheric balloons (inside the magnetosphere) during the last 30–40 years. Among GCR’s, protons and α-particles are largely the most abundant. In addition, there is a lesser amount of nuclei up to nickel, and an even lesser amount of heavier nuclei. Above several GeV/nucleon, the solar modulation slightly affects their energy spectra and, additionally, these latter exhibit a power law behavior as a function of the kinetic-energy per nucleon, $E_k$, e.g., GCR’s intensities are $\sim E_k^{-\gamma}$, where $\gamma$ is the so-called differential spectral index (e.g., see Engelmann et al., 1985; Engelmann, 1990; Mewaldt, 1994; Chapters 5 and 6 of Grieder, 2001; ISO-19390, 2004; Chapter 5 of Stanev, 2004). The spectral index depends on the element, thus the relative abundance may depend on energy. For instance, $\gamma$ is 2.74 (2.60–2.63) for hydrogen (helium) (see Section 5.4.1 of Stanev, 2004). At energies per nucleon...
large enough to neglect the isotopic rest-mass, the same spectral index can be used to describe the kinetic-energy (and total) dependence. Furthermore, relative abundances are shown in Fig. 3 in the paper by Leroy and Rancoita (2007) at 16.2 GeV/nucleon up to nickel (with atomic number \( Z = 28 \)) and in Figs. 7 and 8 by Engelmann et al., 1985 in the energy range from \( \approx 0.8 \) (0.62 for C) up to 35 GeV/nucleon.

The knowledge of the GCR’s abundances can be relevant in determining, for example, the expected radiation effects on human beings and electronics in a space environment. In fact, to a first approximation, since at these kinetic-energies the energy deposition depends on \( Z^2 \) (e.g., see Sections 2.1.1 and 2.1.4 by Leroy and Rancoita, 2009), in such an environment the most abundant nuclei heavier than protons and with masses up to nickel

(i) largely contribute to the overall energy-deposition process inside matter and, thus, to the absorbed dose, i.e., the total ionizing dose (TID) and the displacement dose due to non-ionizing energy-loss (NIEL) processes (e.g., see Sections 3–3.1.5, 3.3–3.3.3 by Leroy and Rancoita, 2007), and

(ii) they have to be taken into account, for instance, in qualification procedures of VLSI components and circuits for space missions (e.g., Colder et al., 2001, 2002a, 2002b; Codegoni et al., 2004a, 2004b; Consolandi et al., 2006; Leroy and Rancoita, 2000, 2007 and references therein).

Moreover, after introducing the radiation weighting factor to account for the biological effectiveness, elements like Fe, Si, Mg and O make even larger contribution to the dose equivalent. Therefore, a realistic evaluation of these effects is needed for long duration space missions, like interplanetary journeys or those for the International Space Station (ISS), particularly for manned missions. Furthermore, for orbital missions with orbits much lower than that geostationary, the local intensities and, thus, relative abundances of GCR’s are also expected to be affected by the Earth’s magnetosphere, where the propagation of CR’s is determined by their rigidities: isotopes with the same kinetic-energy per nucleon can have different rigidities, since they are related to the \( I_{\text{iso}}/Z \) ratio, where \( I_{\text{iso}} \) is the number of nucleons of the nuclide. Thus, these relative abundances are expected to be slightly different among heavy isotopes, but to be largely varied with respect to protons with \( I_{\text{iso}}/Z \approx 1 \). Note that, inside the magnetosphere, the geomagnetic cut-off values for rigidities have been computed since 1985 in the energy range from \( \approx 0.8 \) (0.62 for C) up to 35 GeV/nucleon.

In the present article using a transmission function (TF) approach (e.g., Kudela and Usoskin, 2004; Bobik et al., 2005a, 2005b, 2006), we have determined the primary fluxes of He-nuclei in the same geomagnetic regions of AMS-01 observations, thus allowing a comparison of the observed spectrum and that obtained using the computed TF and the spectrum at 1 AU. In addition, the local abundances inside the Earth’s magnetosphere are determined for p, He, C and Fe (i.e., nuclei among those more abundant in GCR’s) exploiting AMS-01 and HEAO-3-C2 data collected with positive solar magnetic-field polarities.

It has to be remarked that the usage of the TF approach can allow one to determine the isotopic/nuclear abundances in the magnetosphere at any geomagnetic latitude and, thus, any orbit, provided that the CR spectra are determined at 1 AU. Furthermore, it can be adapted to employ different models of the magnetosphere with the respect to that currently used.

2. Observation data at 1 AU

In June 1998, AMS-01 collected data on board of the Space Shuttle Discovery (flight STS-91) at an altitude of \( \approx 380 \) km and with an inclination of 51.7° from the equatorial plane; the angular acceptance was a cone with an aperture of \( \approx 32° \) from the detector axis, which was mostly oriented towards the local zenith during the observation time (a survey of the apparatus and scientific results is available in (Alcaraz et al., 2000a, b; Aguilar et al., 2002). During the 5–6 days of the data taking, AMS-01 observed \( \approx 10^7 \) protons with kinetic-energy between \( 0.22 < E_k < 199.06 \) GeV and \( \approx 10^6 \) α-particles with kinetic-energy \( 0.074 < E_k < 113.61 \) GeV/nucleon.

The HEAO-3-C2 detector, on board of the HEAO-3 satellite launched in September 1979, measured the isotopic composition of the most abundant components of the CR flux with kinetic-energies between 0.8 (0.62 for C) < \( E_k < 35 \) GeV/nucleon., (relative) atomic masses between 7 \( \leq A \leq 56 \), atomic numbers between 4 \( \leq Z \leq 50 \) at an altitude of \( \approx 500 \) km and with an orbit inclination from the equatorial plane of \( \approx 43.6° \) (Engelmann, 1990). The acceptance cone of the HEAO-3-C2 detector was up to \( \approx 28° \) from the axis, which, in turn, was spinning around the direction pointing towards the Sun. The relative atomic mass (A) is the atomic mass in unified atomic mass units \( (u) \). It can be shown (e.g., see Section 3.1 of Leroy and Rancoita, 2009) that for nuclides \( A I_{\text{iso}} \), where \( I_{\text{iso}} \) is the mass number, i.e., the number of nucleons of the nuclide.
For the present publication (Fig. 1), we use the proton and helium data from AMS-01 observations during the solar cycle 23; carbon and iron data are from HEAO-3-C2 observations performed from October 1979 up to June 1980, i.e., during the solar cycle 21. In both periods the solar activity was rising from the minimum to the subsequent maximum and the solar magnetic-field polarity was positive. In addition, HEAO-3-C2 data periods with Forsbush decreases were excluded (Engelmann, 1990). These spectra were recalculated at 1 AU (outside the magnetosphere) by evaluating the effects of the geomagnetic cut-off and detector acceptances on the observation data, following the procedures described in detail by (Alcaraz et al., 2000a, b, c; Aguilar et al., 2002), for protons and helium nuclides, and by Engelmann (1990) for carbon and iron nuclides.

The angular acceptance of the detectors, the altitude and the inclination of the orbits were such that the observations were obtained under almost similar experimental conditions. For instance, note that by inspecting the proton differential fluxes as function of the kinetic-energy, collected with positive solar-polarity in 1998 by AMS-01 (AMS Collaboration, 2000a) and in August 1994 by CAPRICE (Boezio et al., 1999), one can observe slight systematic variations only below \( \approx 0.5 \) GeV (e.g., see Section 4.1.2.3 of Leroy and Rancoita, 2009) for a further comparison among experimental data.

In Fig. 1 the differential (cosmic) fluxes of protons and helium nuclei are the primary spectra derived at 1 AU by AMS-01 collaboration (e.g., see AMS Collaboration, 2000b; 2000c; 2002) and those of carbon and iron nuclei reported in Section 5 of Engelmann (1990) using HEAO-3-C2 observations. The data of the (cosmic) helium fluxes were published as a function of the helium rigidity (see AMS Collaboration, 2000c; 2002) and recalculated as a function of the kinetic-energy of primary \(^4\)He nuclei for Fig. 1. The slight contamination of \(^3\)He in the primary helium nuclei is treated in (Alcaraz et al., 2000c; 2002) and affects marginally such a distribution. Moreover, the transmission function (discussed in Section 3.2) of helium nuclei in the magnetosphere is derived as a function of the helium rigidity and applied to measured fluxes published as a function of the rigidity. The highest lower limit of the kinetic-energy per nucleon is that for Fe observation data, i.e., \( \approx 0.8 \) GeV/nucleon. Note that the value of 0.8 GeV/nucleon corresponds to the average kinetic-energy of nuclei in a bin energy with a lower limit of \( \approx 0.7 \) GeV/nucleon and an upper limit of \( \approx 0.91 \) GeV/nucleon. For C nuclei, the lowest kinetic-energy bin is from 0.55 up to 0.7 GeV/nucleon with an average value of 0.62 GeV/nucleon.

It has to be remarked that the spectral indexes appear to be different at large energies per nucleon, i.e., where the spectra become steeper (e.g., see Fig. 1). Furthermore in the energy region below a few GeV/nucleon, the solar modulation affects the differential energy spectra and, thus, the spectral indexes exhibit some variations (e.g., see Fig. 1), which are A-dependent.

### 2.1. Abundances outside the magnetosphere at 1 AU

In Fig. 2, the relative abundances to protons for He (He/p), C (C/p) and Fe (Fe/p) nuclei with the same kinetic-energy per nucleon, \( E_k \), are shown as function of \( E_k \); for a kinetic-energy of \( \approx 0.8 \) GeV/nucleon, the value of He/p is \( \approx 0.084 \), but ranges between \( \approx 0.047–0.054 \) above \( \approx 0.45 \) GeV/nucleon, while the values of C/p (Fe/p) range from \( \approx 0.0021 \) (0.00017) down (up) to \( \approx 0.0016 \) (0.00021).

This lower limit of the kinetic-energy per nucleon is that for the observation data of the Fe nuclei, as discussed in Section 2. In addition as shown in Fig. 4, this value

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**Fig. 1.** Differential cosmic fluxes of p, He, C and Fe nuclei. The data for p and He nuclei are from (Alcaraz et al., 2000b, 2000c; Aguilar et al., 2002), for C and Fe from paper by Engelmann (1990). These spectra were recalculated at 1 AU (outside the magnetosphere) by evaluating the effects of the geomagnetic cut-off and detector acceptances on the observation data. The curves are to guide the eye.

**Fig. 2.** Relative abundance of He, C and Fe nuclei to proton as a function of the kinetic-energy in units of GeV/nucleon, above 0.8 GeV/nucleon. The data for p and He nuclei are from (Alcaraz et al., 2000b, 2000c; Aguilar et al., 2002), for C and Fe from Engelmann (1990). The curves are to guide the eye.
almost corresponds to the minimal kinetic-energy transferred inside the magnetosphere for He, C and Fe nuclei in the magnetic region 7 (M7).

These abundances were calculated using the observation data shown in Fig. 1 for the He, C and Fe nuclei and an interpolated curve for the proton differential flux spectrum. Note that the overall proton flux calculated with the interpolated curve differs from that obtained summing the experimentally observed fluxes in each kinetic-energy interval (Fig. 1) by about 0.4% in the overall energy-range between 0.22 and 199.06 GeV. The overall flux above 0.8 GeV/nucleon is expected to be slightly increased by the non-observable energy component. Since for these spectra the spectral index is almost constant above \( \approx 10 \) GeV/nucleon, it could be estimated that the largest correction, i.e., for the Fe integral flux, is lower than \( \approx 0.8\% \).

The resulting relative abundances from fluxes of particles above \( 0.8 \) GeV/nucleon are \( Y_{\text{He}/p}^{\text{cos}} = 6.66 \times 10^{-2} \), \( Y_{\text{C}/p}^{\text{cos}} = 1.88 \times 10^{-3} \) and \( Y_{\text{Fe}/p}^{\text{cos}} = 1.78 \times 10^{-4} \), respectively. These latter may differ from those which are derived using abundances at fixed values of kinetic-energy per nucleon, large enough so that the particle is marginally affected by the solar modulation (e.g., see Fig. 3 in (Leroy and Rancoita, 2007) and Table 24.1 of (Yao et al., 2006); in particular, the value of He/p results larger since, as discussed above, up to a few GeV/nucleon its relative abundance is larger than that found at high energies. As a consequence, the overall relative abundances at 1 AU may be affected by the solar modulation and differ from those expected in the interstellar space (e.g., for a discussion see (Beliaev et al., 1996) and references therein).

3. Propagation of GCR’s inside the magnetosphere

At 1 AU the so-called magnetosphere, created in the space region surrounding the Earth by the geomagnetic field and that one carried by the solar wind, provides a partial shield against the penetration of GCR’s down to the Earth surface. Thus, the GCR’s are primary cosmic rays (PCR) reaching the Earth from the outer space. However the flux of PCR’s is only a tiny fraction of the total particle flux observed at the Earth’s surface or at Low Orbits around the Earth. A dominant flux of Secondary Cosmic Rays (SCR’s) is generated by the interactions of PCR’s with the atmosphere (e.g., see Chapter 1 in Grieder, 2001; Section 2.2.2 in Leroy and Rancoita, 2007; Section 4.1.2.5 in Leroy and Rancoita, 2009) and also references therein.

The transport of cosmic rays through the magnetosphere depends on the rigidity \( (P) \) of a charged particle with charge \( Ze \), where \( e \) is the electronic charge, and momentum \( p \), corresponding to a kinetic-energy per nucleon \( E_k \)

\[
P = pc/Ze
\]

with \( c \) the speed of light. As discussed by Bobik et al. (2006) (see also references therein) and, for instance, Stoermer (1930), Fermi (1950), Shea et al. (1965) and Section 6.2.3.1 in Knecht and Shuman (1985), the rigidity of geomagnetic cut-off \( (P_{\text{cut}}) \) varies as a function of the observation location and increases with decreasing the geomagnetic latitude. Besides, all nuclei are positively charged, therefore, due to the Earth magnetic field structure, some incoming directions are preferred (this effect is at the origin of the East–West anisotropy). Primary cosmic rays (PCR) with rigidities lower than the local value of \( P_{\text{cut}} \) cannot reach that observation region. Furthermore, the rigidity spectrum of secondary protons exhibits a bump close to the local value of \( P_{\text{cut}} \), i.e., inside the so-called penumbra region, where only primary protons are expected (Cooke et al., 1991; Bobik et al., 2006).

As discussed in Section 2.2.2 of (Leroy and Rancoita, 2007) since the proton \( (m_p) \) and nucleon \( (m_{\text{nuc}}) \) masses are approximately equal (e.g., see Section 3.1 in Leroy and Rancoita, 2009), the rigidity of an isotope heavier than proton with kinetic-energy per nucleon \( E_k \) is given by

\[
P_{\text{irot}} = (I_{\text{irot}}/Ze) \cdot (E_k + 2 \cdot m_{\text{nuc}} c^2)^{1/2} \approx I_{\text{irot}} \cdot P_{\text{HI}}/Z
\]

where \( P_{\text{HI}} \) is the proton rigidity with the same kinetic-energy \( E_k \), \( Z \) and \( I_{\text{irot}} \) are the atomic number and the number of nucleons (i.e., the mass number) of the isotope, respectively. In Eq. (1), the ratio \( I_{\text{local}}/Z \) for the most abundant stable isotopes (i.e., up to nickel, see Section 2.2 of Leroy and Rancoita, 2007) ranges from 2.0 up to \( \approx 2.3 \) with the exception of \( ^3\text{He} \) for which it is 1.5. As a consequence from an inspection of Eq. (1), less energetic (but heavier than proton) PCR’s can penetrate deeply the magnetosphere: in any location, the associated geomagnetic cut-off rigidity, \( P_{\text{cut}} \), requires a kinetic-energy of the isotope lower than that of a proton and, thus, the isotopic abundances relative to protons are expected to be larger inside the magnetosphere than those at 1 AU, but outside it, and discussed in Section 2.1.

Bobik et al. (2006) described the passage of primary protons through the magnetosphere to the atmosphere by the so-called transmission function (TF), which was determined by back-tracking the proton trajectories to determine those allowed to primaries. The TF allowed one to reproduce the observed AMS-01 data on the Shuttle Orbiter Discovery using the AMS-01 primary proton spectrum at 1 AU, i.e., at the Earth location inside the heliosphere but outside the magnetosphere. In this article, a similar approach is followed for calculating the TF’s (Section 3.2) of He, C and Fe nuclei discussed in Section 2.

3.1. Magnetosphere and AMS-01 geomagnetic regions

To first approximation, the magnetic field close to the Earth surface has a dipole shape, but moving outwards other contributors become important. Among these we
can mention: the several currents (ring current, Birkeland currents and tail currents) due to charged particles trapped inside the magnetosphere, the effect of the reconnection of the interplanetary magnetic field (few nT) and the geomagnetic field at the magnetopause. Moreover the latitudinal dependence is not geographically symmetric, because the Earth magnetic dipole is tilted with respect to the Earth’s rotation axis and shifted from the Earth’s center. In addition, the geomagnetic equator is located on a tilted surface, which is slowly moving with time in relation to the locations of the magnetic poles.

The AMS-01 proton data (Alcaraz et al., 2000b) refer to the ten geomagnetic regions (see Table 1), M; the He data (Alcaraz et al., 2000c) to the three super-regions indicated with SM in Table 1. As already mentioned, the observation period was June 1998. These regions (M and SM) are defined by means of the Corrected Geomagnetic Coordinates (CGM, 2005). CGM coordinates (latitude and longitude) of a point in space are computed by tracing the internal geomagnetic field line through the specified point to the dipole geomagnetic equator, then returning to the same altitude along the dipole field line and assigning the obtained dipole latitude and longitude as the CGM coordinates to the starting point. It is described by the IGRF model, i.e., the DGRF 2000–2005 model, see Barton (1997), IGRF (2005) and references therein; see also Section 3.2.

3.2. Transmission function and AMS-01 He data

As discussed in Appendix A by Bobik et al. (2006), the magnetic field of the Earth’s magnetosphere is described using

(i) the International Geomagnetic Reference Field (IGRF) 2000–2005 (Barton, 1997) for representing the main contribution due to the inner Earth, and

(ii) the external magnetic field model implemented by Tsyganenko and Stern (called T96-model, see Tsyganenko (1995), Tsyganenko and Stern (1996)) for representing the other current contributions in the magnetosphere.

The International Geomagnetic Reference Field (IGRF) model is the empirical representation of the Earth’s magnetic field recommended for scientific use by the International Association of Geomagnetism and Aeronomy (IAGA) (e.g., see IGRF (2005)). As already mentioned, the IGRF model is employed to evaluate the main (core) field without external sources and consists of a series of mathematical models describing the Earth’s main field and its secular variation.

In the region inside the magnetopause, the T96-model calculates the external magnetic field in every point of the space out to 70 Re in magnetosphere tail (where Re is the radius of the Earth). This field component is generated by charged particles circulating outside the solid Earth body. The external field component takes into account the interaction with the solar wind. The more complex morphology of the field and the variability of the solar activity make this one as the critical component. The model includes the following implementations: the position and the shape of the magnetopause, the boundary surface of the magnetosphere, is explicitly defined; the magnetic field of the region around the magnetopause is also considered; the interconnection of the Earth magnetosphere to the solar wind field at the boundary is taken into account too; furthermore the magnetic fields generated by the regions 1 and 2 of Birkeland currents, by the ring current, and by the tail current are added. The Earth’s magnetopause is calculated using the Sibeck equation (Sibeck et al., 1991) modified by Tsyganenko (Tsyganenko, 1995) for the solar wind effect. We have introduced an empirical magnetospheric boundary large 25 Re in the night-side region to avoid long calculations in the far tail.

Access for PCR to some place is supposed to be allowed when the back-traced particle trajectory reaches the magnetopause or the magnetospheric boundary. As internal boundary we have considered a sphere at an altitude of 40 km, corresponding to the surface containing the 99% of the Earth atmosphere. Thus using the time-dependent code discussed in Appendix A of (Bobik et al. (2006)), the TF requires the determination of the so-called allowed trajectories of the particles entering the AMS-01 or HEAO-3 spectrometers, following a back-tracking procedure. The locations (3600) of the particles to be back-tracked are distributed uniformly over a complete sphere surrounding the Earth at an altitude of 400 km, excluding the South-Atlantic anomaly region (i.e., the region with latitude between −55° and 0° and with longitude between −80° and 20°). Charged particles are generated at the position of the space detectors AMS-01. They are back-tracked in time until they reach one of the two boundaries: the magnetopause/magnetospheric boundary or the atmosphere. Some reconstructed trajectories are terminated after a selected number of steps to avoid long calculations for the trajectories of trapped particles. In the first case the particles are considered to be primary CR, otherwise secondary CR. The external field is evaluated taking into account the parameters changing with the solar activity. Those parameters are evaluated at the time of the data taking of the two experiments.

<table>
<thead>
<tr>
<th>Region (M)</th>
<th>Super-region (SM)</th>
<th>CGM latitude $\theta _{M}$</th>
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<tr>
<td>1</td>
<td>a</td>
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<td>2</td>
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<tr>
<td>9</td>
<td>c</td>
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<tr>
<td>10</td>
<td>c</td>
<td>$1.0 \leq</td>
</tr>
</tbody>
</table>
Initially, for the He data of AMS-01 experiment, the particle directions (27°) are isotropically distributed within the outward hemisphere and inside the 32° acceptance cone (around the local geocentric Zenith) of the AMS-01 spectrometer. The TF was computed for the same 31 rigidity intervals of the AMS-01 data (see Table 1 of paper by AMS Collaboration (2000c)), i.e., the lowest rigidity bin is about 0.76–0.91 GV and the highest about 190.55–229.09 GV. Within the acceptance cone of the AMS-01 spectrometer, about $2.3 \times 10^8$ particle trajectories were reconstructed. For the three geomagnetic super-regions, the TFSM has been averaged over all uniformly distributed geographic-locations and weighted to take into account the observation time spent in each region by the flying detector:

$$TF_{SM}(P_{b,He}) = \frac{\sum_i N^{all}_{b,He,i}}{\sum_i N^{total}_{b,He,i}}$$

where $P_{b,He}$ is the He rigidity in the $b$th rigidity interval of width $\Delta P_{b,He}$, $TF_{SM}(P_{b,He})$ is the transmission function for the position $i_{SM}$ inside the geomagnetic region SM and $\sum_i N^{total}_{b,He,i}$ is the total number of locations for the same region. For the location $i_{SM}$ the $TF_{SM}(P_{b,He})$ is given by:

$$TF_{SM}(P_{b,He,i}) = \frac{N^{all}_{b,He,i} - N^{forb}_{b,He,i}}{N^{total}_{b,He,i}}$$

where $N^{all}_{b,He,i}$ is the number of allowed trajectories and the total (allowed and forbidden) number of computed trajectories is

$$N^{total}_{b,He,i} = N^{all}_{b,He,i} + N^{forb}_{b,He,i}$$

The ratio $TF_{SM}(P_{b,He,i})$ represents the probability that particles with rigidity $P_{b,He}$ reach the geographic position $i_{SM}$ coming from outside the magnetosphere. Then, for every geomagnetic super-region, SM, the averaged ratio $TF_{SM}(P_{b,He})$ over all the positions $i_{SM}$ expresses the transmission function for a particle with rigidity $P_{b,He}$ to reach that geomagnetic super-region SM, at the altitude of AMS. Each starting position and direction has been chosen in order to have a uniformly distributed sample.

Following the procedure already discussed by Bobik et al. (2006), in each super-region SM the primary flux $\phi^{p}_{SM}(P_{b,He})$ of He-particles with rigidity $P_{b,He}$ can be evaluated using the cosmic ray flux $\phi^{cos}(P_{b,He})$ from Table 1 of the paper by Alcaraz et al. (2000c):

$$\phi^{cos}_{SM}(P_{b,He}) = \phi^{cos}(P_{b,He}) \cdot TF_{SM}(P_{b,He,i})$$

The experimental He-fluxes measured by AMS-01 and those calculated using Eq. (4) are in good agreement and are shown in Fig. 3.

### 3.3. Transmission functions and fluxes inside the M-regions

Using the procedure described in Section 3.2, the TF’s for $p$, He, C and Fe nuclei were computed for the ten geomagnetic regions M (see Table 1) at an altitude of 400 km and inside a 28° acceptance cone (around the local geocentric Zenith), i.e., the HEAO-3 acceptance cone which was the smallest of the two spectrometers. For protons (as for helium) the simulated particles were $2.3 \times 10^8$, for C and Fe nuclei $10^7$. Note that in Bobik et al. (2006), the TF for protons was computed for the AMS-01 acceptance cone. The TF’s are shown in Fig. 4 for the three regions M1, M4 and M7 as function of the kinetic-energy in GeV for protons and the kinetic-energy in GeV/nucleon for He, C and Fe nuclei. Since the TF’s are evaluated using the particle rigidities, the corresponding kinetic-energies were computed assuming a negligible $^3$He contamination in the He-flux of AMS-01 observations (e.g., see Section 2) and, similarly, C and Fe fluxes consisting mainly of $^{12}$C and $^{56}$Fe isotopes. The TF becomes 0, when incoming PCR’s cannot reach the observation location any more, thus the particle rigidity is lower than the local rigidity of geomagnetic cut-off ($P_{cml}$). It is 1 when the full flux of PCR’s can reach that location. The penumbra region corresponds to TF-values larger than 0 and lower than 1. The geomagnetic cut-off rigidity almost similarly affects the propagation of $p$, He, C and Fe nuclei inside the magnetosphere, but as already discussed, for nuclei the corresponding kinetic-energy per nucleon depends on the $I_{sol}/Z$ ratio (Eq. (1)).

In Fig. 4, it can be observed the expected shift of the kinetic-energies per nucleon, at which the TF becomes 0 or 1 with respect to the kinetic-energies for protons. As already mentioned, this effect arises because the ratio $I_{sol}/Z$ for the most abundant nuclei in GCR’s (e.g., see Section 2.2 of Leroy and Rancoita (2007) and references therein), except for protons for which this ratio is 1. In fact, the transition to a complete propagation inside the magnetosphere (i.e., $TFM = 1$) occurs at larger energies for pro-
tons with respect to those for isotopes with atomic number $Z \gg 2$ (Fig. 4). Furthermore, He, C and Fe nuclei with kinetic-energies $\approx 0.8 \text{ GeV/nucleon}$ can penetrate the Earth’s magnetosphere up the region M7. In the geomagnetic region M7, at 0.8 GeV/nucleon, the TF is 0.14, 0.11 and 0.16 for He, C and Fe nuclei (see Fig. 4). As a consequence, for observation data with kinetic-energies larger than $0.8 \text{ GeV/nucleon}$ (i.e., that corresponding to lower limit for Fe observation data, as discussed in Section 2), only the TF’s for the magnetic regions M1–M7 can be determined for all nuclei, i.e., including iron.

Using those so calculated TF’s, the PCR’s fluxes at an altitude of 400 km are obtained from the AMS-01 cosmic flux (i.e., at 1 AU) for p and He and HEAO-3-C2 flux for C and Fe. If $\Phi_M^{\text{pri}}(E_k)$ is the transmission function for a particle with kinetic-energy per nucleon $E_k$, the primary flux $\Phi_M^{\text{pri}}(E_k)$ in the region M is given by

$$\Phi_M^{\text{pri}} = \Phi^{\text{cos}}(E_k) \cdot TF_M(E_k)$$

where $\Phi^{\text{cos}}(E_k)$ is the cosmic ray flux of particles with kinetic-energy per nucleon $E_k$. In Fig. 5 the fluxes of p, He, C and Fe nuclei are shown for the regions M1 and M7, as an example.

The above computed TFs are valid for quiet time conditions. During the strong geomagnetic disturbances their values can be obtained using various geomagnetic field models providing different predictions of transmissivity (Kudela et al., 2008).

### 3.4. Relative abundances inside the magnetosphere

In the seven magnetic regions (M) for which the TF can be properly calculated, the abundances of PCR’s relative to proton are determined using the primary fluxes in Eq. (5) as:

$$R_{\text{He/p}}(M) = \frac{\int \Phi_M^{\text{pri}}(E'_k) dE'_k}{\int \Phi_M^{\text{pri}}(E_k) dE_k}$$

(6)

$$R_{\text{C/p}}(M) = \frac{\int \Phi_M^{\text{pri}}(E'_k) dE'_k}{\int \Phi_M^{\text{pri}}(E_k) dE_k}$$

(7)

$$R_{\text{Fe/p}}(M) = \frac{\int \Phi_M^{\text{pri}}(E'_k) dE'_k}{\int \Phi_M^{\text{pri}}(E_k) dE_k}$$

(8)

Note that $\Phi_M^{\text{pri}}(E_k) = 0$ below the penumbra region in each geomagnetic region M, i.e., for $TF_M(E_k) = 0$ as discussed in Section 3.3.

As already mentioned in Section 2, the high energy component of the isotopic spectra contributes to the integral flux by less than 0.8% in case of iron (i.e., where such a correction is largest). However, the effect of the geomagnetic cut-off rigidity depletes the particle flux with increasing the geomagnetic latitudes. Thus the correction due to the high energy component may become more relevant and, as already mentioned (Section 2.1), it can be estimated assuming that for these spectra the spectral indexes are almost constant above $\approx 10 \text{ GeV/nucleon}$. The resulting corrections are $\approx 0.8\%$, $0.8\%$, $3.9\%$ and $7.7\%$ for the proton, He, C and Fe fluxes in the geomagnetic region M1, where they are the largest. These corrections are taken into account in determining the ratios $R_{\text{He/p}}, R_{\text{C/p}}, R_{\text{Fe/p}}$ shown in Table 2.

It has to be remarked that, in particular for the geomagnetic region M7, the TF allows Fe nuclei with kinetic-energy per nucleon slightly lower than 0.8 GeV/nucleon to penetrate inside the magnetosphere (e.g., see Fig. 4).

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Fig. 4. Transmission function of p, He, C and Fe for the geomagnetic regions M1 (dot line), M4 (dash-dot-dot line) and M7 (dash line).

Fig. 5. Flux of primaries p, He, C and Fe, in the geomagnetic regions M1 (circles), and M7 (triangles). The lines are to guide the eye.
but HEAO-3-C2 observation data were collected for Fe nuclei with average kinetic-energies per nuclei larger than 0.8 GeV/nucleon for the lowest energy-bin (Section 2). However in the region M7, \( \delta R_{Fe/p} \) and \( \delta R_{C/p} \) are decreased with respect to those reported in Table 2 by about (0.7– 0.8)%., when the fluxes only account for particles with kinetic-energies larger than 0.7 GeV/nucleon, which corresponds to the lower limit of that observed for Fe nuclei. Thus, although the fluxes of particles are large for particle nuclei with kinetic-energies higher than 0.8 GeV/nucleon, which correspond to the threshold kinetic-energy of the observation data. The ratios \( R(M) \) and the enhancement factors \( \Delta(M) = \frac{R(M)}{R_{Hall}} \), where the values of \( R^{cos} \) were derived in Section 2.1, are shown in Table 2 for geomagnetic regions M1– M7. The enhancement factors account for the increase of isotopic abundances inside the magnetosphere with respect to the modulated values outside it at 1 AU. From an inspection of Table 2, it is possible to note that the ratios \( \Delta(M) \)

(i) depend on the mass number \( I_{not} \),
(ii) range from \( \approx 2.06 \) up to \( \approx 3.74 \) in the seven investigated geomagnetic regions and, finally,
(iii) exhibit a slight dependence on the geomagnetic latitude of up the geomagnetic region M7: in the first six regions \( \Delta \) varies from the average by less than \( \pm 4.8, 6.9 \) and 13.5% for He/p, C/p and Fe/p, respectively. On average in the seven geomagnetic regions, the enhancements factors are \( \approx 2.31, \approx 2.71 \) and \( \approx 3.35 \) for He/p, C/p and Fe/p, respectively. These factors also account for the variation of the ratio \( I_{not}/Z \) with the mass number and the overall behavior of the spectral indexes with the kinetic-energy for the modulated spectra.

### 4. Conclusion

At 1 AU and outside the Earth’s magnetosphere, the relative abundances to protons for He (He/p), C (C/p) and Fe (Fe/p) nuclei were calculated using the observation data of AMS-01 (for p and He) and HEAO-3 (for C and Fe) above 0.8 GeV/nucleon. The relative abundances account for the modulation of GCR’s inside the heliosphere and are \( R_{He/p}^{cos} \approx 6.66 \times 10^{-2}, R_{C/p}^{cos} \approx 1.88 \times 10^{-3} \) and \( R_{Fe/p}^{cos} \approx 1.78 \times 10^{-3} \) for He/p, C/p and Fe/p, respectively. These values may differ from those which are derived using abundances fixed values of kinetic-energy per nucleon, large enough so that particles are marginally affected by the solar modulation.

The penetration depth of PCR’s inside the magnetosphere depends on their rigidity and observation location, thus on the geomagnetic cut-off rigidity. The rigidity almost similarly affects the propagation of p, He, C and Fe nuclei through the magnetosphere, but for nuclei the corresponding kinetic-energy per nucleon depends on the \( I_{not}/Z \) ratio which differs by a factor almost 2 from that of protons. As a consequence, the relative isotopic abundances with respect to protons are expected to be modified with respect to those at 1 AU but outside the magnetosphere.

An approach using the back-tracking of simulated particles was used to define the probability, i.e., the transmission function, that a primary proton, helium, carbon and iron nucleus can reach the magnetopause from the observation location. Consequently, it has allowed one to describe the transport properties of the PCR’s to the geomagnetic regions of the AMS-01 observations (at an altitude of about 400 km) from the upper limit of the geomagnetic field, i.e., the magnetopause.

The fluxes of primary He-nuclei calculated by means of the so derived transmission functions exhibit a good agreement with the spectra observed in three magnetic (super-) regions by AMS-01 experiments. Furthermore, the relative abundances and their ratios with those outside the magnetosphere at 1 AU (called enhancement factors) were also computed for the magnetic regions up to M7. For geomagnetic latitudes ranging from \( \approx -45.84^\circ \) up to \( \approx 45.84^\circ \), the enhancement factors, accounting for the variation of the relative isotopic abundances, depend on the mass number \( I_{not} \) and, on average, range from \( \approx 2.31 \) up to \( \approx 3.35 \). Thus, the magnetospheric isotopic relative abundances result to be modified because the proton component is more depleted than the other isotopic components of GCR’s...
and differ from those inside the solar cavity and those in the interstellar space.

It has to be remarked that, for any acceptance angle, the usage of the TF approach can allow one to determine the isotopic/nuclear abundances in the magnetosphere at any geomagnetic latitude and, thus, any orbit, provided that the CR spectra are determined at 1 AU.

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References